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# Electromagnetic suppression of the decay $\mu \rightarrow e\gamma$

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## Abstract

Due to large QED anomalous dimensions of the electric and magnetic dipole operators, the rate of the rare muon decay  $\mu \rightarrow e\gamma$  is suppressed by the factor  $\left(1 - \frac{8\alpha}{\pi} \ln \frac{\Lambda}{m_\mu}\right)$ , independent of the physics responsible for the lepton-flavor violation, except for the scale  $\Lambda$  at which it occurs. For  $\Lambda = 100 \dots 1000$  GeV, the resulting decrease of the rate amounts to about 12 ... 17%.

# 1 Introduction

The only observed decay channel of the muon is  $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$  (with possible photon or electron-positron pair emission). However, since the discovery of the muon more than half century ago, searches have been undertaken for the decay  $\mu \rightarrow e\gamma$ . Initially, when the muon was thought to be an excited state of the electron, this was expected to be its dominant decay channel. It was soon realized that it is very strongly suppressed (the early experiments are summarized in [1]). When an intermediate boson was proposed to explain the mechanism of weak interactions [2], the absence of  $\mu \rightarrow e\gamma$  led to the hypothesis that the two neutrinos in the muon decay (Fig. 1(a)) have different flavors so that the interaction shown in Fig. 1(b) cannot occur [3, 4]. The existence of the muon neutrino, distinct from the electron one,

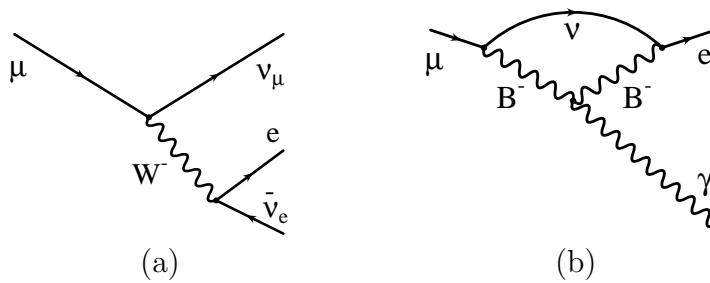


Figure 1: (a) Ordinary muon decay; (b) The puzzle of  $\mu \rightarrow e\gamma$  absence in the early models with an intermediate vector boson.

was demonstrated in the classic 1962 experiment in Brookhaven [5]. In this way, the limits placed on the branching ratio for  $\mu \rightarrow e\gamma$  helped establish the concept of families or generations of fermions, which became one of the cornerstones of the standard model.

In fact, the standard model with massless neutrinos strictly forbids the lepton-flavor nonconserving transitions like  $\mu \rightarrow e\gamma$ . Even if the neutrinos have a small mass, the rate is still very small,  $\mathcal{O}((m_\nu/m_W)^4)$  [6, 7, 8, 9]. However, most extensions of the standard model, containing some new physics at the hitherto unexplored mass scales, predict a higher rate of  $\mu \rightarrow e\gamma$ . For example, in supersymmetry (SUSY) neutrinos have heavy “partners”, scalar sneutrinos, whose mixing could generate  $\mu \rightarrow e\gamma$  transitions through the interaction with charginos  $\tilde{\chi}^\pm$ , as shown in Fig. 2(a). Scalar partners of the charged leptons, interacting with neutralinos  $\tilde{\chi}^0$ , could also contribute to this decay (Fig. 2(b)).

Explicit supersymmetric grand unified models [10, 11, 12, 13, 14] predict a  $\mu \rightarrow e\gamma$

rate just below the present 90% CL bound from the MEGA experiment, [15],

$$\frac{\Gamma(\mu \rightarrow e\gamma)}{\Gamma(\mu \rightarrow e\bar{\nu}_e\nu_\mu)} < 1.2 \cdot 10^{-11}. \quad (1)$$

In the near future, a new search for  $\mu \rightarrow e\gamma$  will be undertaken at the Paul Scherrer Institute (PSI) [16], with a single event sensitivity corresponding to the branching ratio of  $2 \times 10^{-14}$ . In view of the SUSY GUT predictions, it is not inconceivable that this experiment will find  $\mathcal{O}(100)$  of  $\mu \rightarrow e\gamma$  decay events. At such rate, precision studies of lepton-number violating interactions will become possible. It is therefore interesting to theoretically evaluate model-independent electromagnetic effects which turn out to decrease the rate of  $\mu \rightarrow e\gamma$  by several percent.

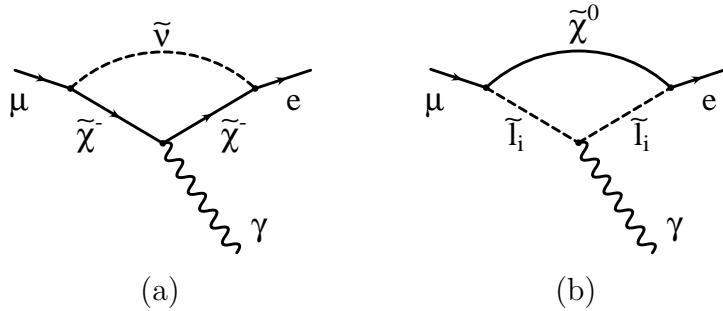


Figure 2: Supersymmetric amplitudes which might give rise to the decay  $\mu \rightarrow e\gamma$ .

## 2 QED suppression of the dipole operators

The effective interaction which gives rise to  $\mu \rightarrow e\gamma$  has the form

$$= \bar{e} \sigma^{\mu\nu} (f_M + f_E \gamma_5) \mu \cdot q_\mu A_\nu, \quad (2)$$

where  $f_i$  ( $i = M, E$ ) are formfactors, calculable in explicit models of physics beyond the standard model. In terms of  $f_i$ , the rate of  $\mu \rightarrow e\gamma$  is

$$\Gamma^{(0)}(\mu \rightarrow e\gamma) = \frac{m_\mu^3}{8\pi} \left( |f_M|^2 + |f_E|^2 \right). \quad (3)$$

It is well known that the chirality-flipping electric and magnetic dipole operators in (2) have (the same) large QED anomalous dimension. It was first computed in the context of hadron decays in QCD [17, 18, 19, 20], and plays an important role in various electromagnetic processes like the radiative decay  $b \rightarrow s\gamma$  [21] or the muon anomalous magnetic moment [22, 23, 24] (see also [25]).

We denote the coefficient of the dipole-transition operators in (2), computed in a full theory violating lepton flavor, by  $f_i(\Lambda)$ , where  $\Lambda$  is a characteristic mass scale of the relevant new physics. For example, in SUSY,  $f_i(\Lambda)$  would result from the one-loop diagrams in Fig. 2, and  $\Lambda$  would be the characteristic mass of the superpartners. If we now consider an effective theory at an energy of the order of the muon mass, the heavy exotic fields are not dynamical degrees of freedom and we can consider the effects of Fig. 2 as point-like interactions given by the Lagrangian (2).

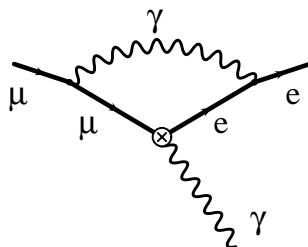


Figure 3: An example of an electromagnetic correction which contributes to the suppression of the  $\mu \rightarrow e\gamma$  decay rate.

However, when we consider higher-order electromagnetic corrections to this interaction, such as the one shown in Fig. 3, we find that they are logarithmically divergent in the ultraviolet (UV). This is not surprising, since the dimension of the operators in (2) is 5, which signals non-renormalizability. An explicit calculation shows that the effect of those corrections amounts to

$$f_i(\Lambda) \rightarrow f_i(\Lambda) \left( 1 - \frac{4\alpha}{\pi} \ln \frac{\Lambda}{m_\mu} + \mathcal{O}(\alpha) \right), \quad (4)$$

where we have taken the UV cut-off to be equal  $\Lambda$ , since around that magnitude of the loop momentum it is no longer justified to treat the flavor-changing vertex as point-like. The interaction is weakened; we can denote its effective strength at the muon mass scale by  $f_i(m_\mu)$ , which includes the leading logarithmic effect,

$$f_i(m_\mu) = f_i(\Lambda) \left( 1 - \frac{4\alpha}{\pi} \ln \frac{\Lambda}{m_\mu} \right). \quad (5)$$

This effect can be quite large, since the rate (3) of the decay is proportional to the sum of squares of  $f_i$ ,

$$\Gamma(\mu \rightarrow e\gamma) \simeq \left(1 - \frac{8\alpha}{\pi} \ln \frac{\Lambda}{m_\mu}\right) \Gamma^{(0)}(\mu \rightarrow e\gamma). \quad (6)$$

If  $\Lambda$  is of order 250 GeV, which is a typical SUSY mass scale in the models considered in [10], this corresponds to about 14% decrease of the rate.

It is possible to sum up the leading log effects to all orders in  $\alpha^n \ln^n \Lambda/m_\mu$  (see e.g. [26, 27]). In the absence of mixing with other lepton-flavor non-conserving operators, the scale dependence of the coefficients  $f_i$  can be expressed in an iterative form,

$$f_i(m_<) = f_i(m_>) \cdot \left( \frac{\alpha(m_<)}{\alpha(m_>)} \right)^{\gamma/b}, \quad (7)$$

where in our case the anomalous dimension is  $\gamma = -8$  and  $b$  is determined using the charges  $Q_j$  of all particles contributing to the running of the fine structure constant between the scales  $m_<$  and  $m_>$ :

$$b = -\frac{4}{3} \sum_j Q_j^2. \quad (8)$$

The explicit result for  $f_i(m_\mu)$  depends on the mass spectrum of a concrete new physics scenario. However, higher order leading-logarithmic effects are not expected to significantly change the magnitude of the  $\mu \rightarrow e\gamma$  rate decrease given in (6), because of cancelation between the running of the fine structure constant and the effects of higher orders in the anomalous dimension. Similar cancelation was observed in the muon  $g - 2$  [24].

Typical lepton-flavor violating amplitudes, like the ones in Fig. 2, contain two new physics masses, which in general may be quite different. One can ask the question, what should be taken as the argument  $\Lambda$  of the logarithm in (6). As long as the ratio of the two large scales is small compared to their size relative to the muon mass, this is an issue of non-leading corrections, which we have been neglecting. In the case of  $\mu \rightarrow e\gamma$  induced by the small neutrino masses (where the rate is extremely small, as discussed above), the scale  $\Lambda = m_W$  in (6) is the larger of the two masses in the loop. The inverse of  $m_W$  determines the size of the effective interaction range.

### 3 Four-fermion operators

New physics effects can also induce lepton-flavor violating four-fermion operators such as  $(\bar{e}\Gamma\mu)(\bar{f}\Gamma f)$  (Fig. 4(a)). They contribute to  $\mu \rightarrow e\gamma$  through loop effects

(Fig. 4(b,c)) in the same order in  $\frac{\alpha}{\pi} \ln \frac{\Lambda}{m_\mu}$  as the suppression effect in eq. (6).

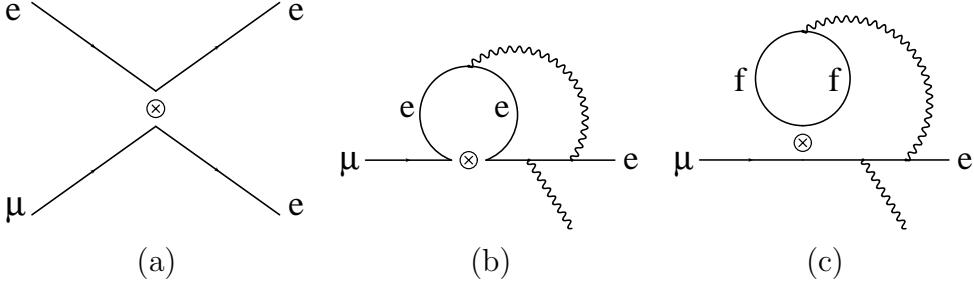


Figure 4: (a) Lepton flavor-violating four-fermion operator; (b) Example of a contribution to  $\mu \rightarrow e\gamma$  for  $f = e$  or  $\mu$ ; (c) Example of other fermions' contribution.

In theories such as  $R$ -parity conserving SUSY, four-fermion contributions are suppressed relative to the dipole operators (Fig. 2) by two powers of a coupling constant and are not expected to contribute significantly to  $\mu \rightarrow e\gamma$ . It is, however, interesting to see to what extent we can estimate such contributions in a model-independent way.

Virtual fermions  $f$  other than muon or electron contribute only through “closed” loops, as shown in Fig. 4(c). Large logarithms arising from such diagrams cancel at least partially in anomaly-free theories, and we will neglect these effects.

Here we will consider a specific example of the operator

$$\mathcal{O}_x = G_x (\bar{e} \gamma^\nu L \mu)(\bar{e} \gamma_\nu L e), \quad L \equiv \frac{1 - \gamma_5}{2}, \quad (9)$$

whose anomalous dimension and mixings with other flavor-violating operators can be found using well-known results found in studies of the radiative quark decay  $b \rightarrow s\gamma$ . We will demonstrate that the bound on  $G_x$  obtained from searches for  $\mu \rightarrow eee$  renders the contribution of this operator to  $\mu \rightarrow e\gamma$  negligible. We may expect that contributions of other Dirac structures and of operators  $(\bar{e} \Gamma \mu)(\bar{\mu} \Gamma \mu)$  have similar magnitudes.

Operator  $\mathcal{O}_x$  induces the decay  $\mu \rightarrow eee$  with a rate

$$\Gamma(\mu \rightarrow eee) = \frac{G_x^2 m_\mu^5}{768 \pi^3}, \quad (10)$$

and we can use the bound on the branching ratio [28],

$$\frac{\Gamma(\mu \rightarrow eee)}{\Gamma(\mu \rightarrow e\nu\nu)} < 10^{-12}, \quad (11)$$

to constrain  $G_x$ . We find

$$G_x < 2 \cdot 10^{-6} G_F, \quad (12)$$

( $G_F$  is the Fermi constant [28]).

In order to find the contribution of  $\mathcal{O}_x$  to the amplitude  $\mu \rightarrow e\gamma$  we consider its mixing with the dipole operators in eq. (2). We write the result as

$$g_x \bar{e} \sigma^{\mu\nu} (1 + \gamma_5) \mu \cdot q_\mu A_\nu, \quad (13)$$

with

$$g_x = \frac{em_\mu G_x}{16\pi^2} \frac{29\alpha}{18\pi} \ln \frac{\Lambda}{m_\mu}, \quad (14)$$

where  $e = \sqrt{4\pi\alpha} \simeq 0.3$ . Finally, we would like to compare the effect of this four-fermion operator on the form-factors  $f_i$  ( $i = E, M$ ) with the effect of the QED correction in eq. (4). For this purpose we assume  $f_E = f_M$  and consider the quantity

$$R = \frac{g_x}{f_i \frac{4\alpha}{\pi} \ln \frac{\Lambda}{m_\mu}} < e \frac{29\sqrt{3}}{8\pi \cdot 18} \cdot 10^{-6} \cdot \frac{1}{\sqrt{\text{BR}(\mu \rightarrow e\gamma)}}, \quad (15)$$

where we have taken  $f_i = \frac{G_F m_\mu}{4\sqrt{3}\pi} \sqrt{\text{BR}(\mu \rightarrow e\gamma)}$  and used the bound (12). If  $\mu \rightarrow e\gamma$  is discovered with a branching ratio between  $10^{-11}$  and  $10^{-14}$ , the upper bound on the ratio  $R$  of the four-fermion and dipole radiative effects will be between about  $10^{-2}$  and 0.3.

The QED corrections we considered in this paper will be relevant for the upcoming PSI experiment if it observes a fair number (of the order of a hundred or more) of decay events  $\mu \rightarrow e\gamma$ . This corresponds to the branching ratio of at least  $10^{-12}$ , for which the ratio  $R$  is about 0.03. We conclude that the effects of the four-fermion operators are likely to be negligible for the next generation of the  $\mu \rightarrow e\gamma$  searches.

## 4 Conclusions

The logarithmic suppression which we have discussed in Section 2 affects not only  $\mu \rightarrow e\gamma$  but also other lepton-flavor violating processes occurring via the dipole transition of the type (2). For example, the rates of the  $\tau$ -lepton decays  $\tau \rightarrow \mu\gamma$  and  $\tau \rightarrow e\gamma$  are decreased by

$$1 - \frac{8\alpha}{\pi} \ln \frac{\Lambda}{m_\tau}, \quad (16)$$

which is about  $7.5 \dots 12\%$  for  $\Lambda = 100 \dots 1000$  GeV. On the other hand, the decays of the type  $\mu^+ \rightarrow e^+e^+e^-$  and muon-electron conversion in the nuclear field,  $\mu^-N \rightarrow e^-N$ , can occur via a more general interaction, including monopole formfactors, which do not receive such logarithmic corrections.

To summarize, we have pointed out an electromagnetic short-distance effect which decreases the predicted rate of the lepton-flavor violating decay  $\mu \rightarrow e\gamma$  by a factor  $(1 - \frac{8\alpha}{\pi} \ln \frac{\Lambda}{m_\mu})$ , or  $12 \dots 17\%$  for the new physics scale  $\Lambda = 100 \dots 1000$  GeV. If the lepton-flavor non-conservation is observed by the next generation of experiments, the  $\mu \rightarrow e\gamma$  search at the PSI and the conversion  $\mu^-N \rightarrow e^-N$  search MECO in Brookhaven, this correction will help disentangle the underlying new physics structure.

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